Paper Reference(s)

6663/01

Edexcel GCECore Mathematics C1

Gold Level G4

Time: 1 hour 30 minutes

Materials required for examination

Items included with question

papers

Mathematical Formulae (Green)

Nil

Candidates may use any calculator allowed by the regulations of the Joint

Council for Qualifications. Calculators must not have the facility for symbolic

algebra manipulation, differentiation and integration, or have retrievable

mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C1), the paper reference (6663), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

There are 11 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	В	C	D	E
60	52	43	34	25	16

1. (a) Find the value of $16^{-\frac{1}{4}}$.

(2)

(b) Simplify $x \left(2x^{-\frac{1}{4}}\right)^4$.

(2)

January 2011

2. Given that $32\sqrt{2} = 2^a$, find the value of a.

- **(3)**
- June 2009
- 3. Show that $\frac{2}{\sqrt{12-\sqrt{8}}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers.
- **(5)**
- **May 2012**

- $\frac{\mathrm{d}y}{\mathrm{d}x} = 5x^{-\frac{1}{2}} + x\sqrt{x}, \qquad x > 0.$
 - Given that y = 35 at x = 4, find y in terms of x, giving each term in its simplest form.
- **(7)**
- January 2010

5.

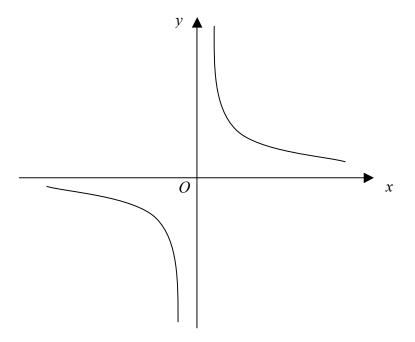


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \frac{3}{x}$, $x \neq 0$.

(a) On a separate diagram, sketch the curve with equation $y = \frac{3}{x+2}$, $x \ne -2$, showing the coordinates of any point at which the curve crosses a coordinate axis.

(3)

(b) Write down the equations of the asymptotes of the curve in part (a).

(2)

May 2007

6. The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots.

Find the value of p.

(4)

June 2009

7. $f(x) = x^2 + (k+3)x + k$

where *k* is a real constant.

(a) Find the discriminant of f(x) in terms of k.

(2)

(b) Show that the discriminant of f(x) can be expressed in the form $(k + a)^2 + b$, where a and b are integers to be found.

(2)

(c) Show that, for all values of k, the equation f(x) = 0 has real roots.

(2)

May 2011

8. $4x - 5 - x^2 = q - (x + p)^2,$

where p and q are integers.

(a) Find the value of p and the value of q.

(3)

(b) Calculate the discriminant of $4x - 5 - x^2$.

(2)

(c) Sketch the curve with equation $y = 4x - 5 - x^2$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(3)

May 2012

9. The first term of an arithmetic series is a and the common difference is d.

The 18th term of the series is 25 and the 21st term of the series is $32\frac{1}{2}$.

(a) Use this information to write down two equations for a and d.

(2)

(b) Show that a = -17.5 and find the value of d.

(2)

The sum of the first *n* terms of the series is 2750.

(c) Show that n is given by

$$n^2 - 15n = 55 \times 40.$$

(4)

(*d*) Hence find the value of *n*.

(3)

January 2009

10.

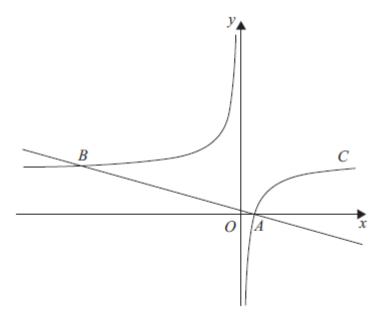


Figure 2

Figure 2 shows a sketch of the curve C with equation

$$y=2-\frac{1}{x}, \quad x\neq 0.$$

The curve crosses the x-axis at the point A.

(a) Find the coordinates of A.

(1)

(b) Show that the equation of the normal to C at A can be written as

$$2x + 8y - 1 = 0. (6)$$

The normal to C at A meets C again at the point B, as shown in Figure 2.

(c) Find the coordinates of B.

(4)

January 2012

11. The curve C has equation

$$y = x^3 - 2x^2 - x + 9$$
, $x > 0$.

The point P has coordinates (2, 7).

(a) Show that P lies on C.

(1)

(b) Find the equation of the tangent to C at P, giving your answer in the form y = mx + c, where m and c are constants.

(5)

The point Q also lies on C.

Given that the tangent to C at Q is perpendicular to the tangent to C at P,

(c) show that the x-coordinate of Q is $\frac{1}{3}(2+\sqrt{6})$.

(5)

June 2009

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	$16^{\frac{1}{4}} = 2$ or $\frac{1}{16^{\frac{1}{4}}}$ or better	M1
	$\left(16^{-\frac{1}{4}} = \right) \frac{1}{2} \text{ or } 0.5$	A1 (2)
(b)	$\left(2x^{-\frac{1}{4}}\right)^4 = 2^4 x^{-\frac{4}{4}} \text{ or } \frac{2^4}{x^{\frac{4}{4}}} \text{ or equivalent}$	M1
	$x\left(2x^{-\frac{1}{4}}\right)^4 = 2^4 \text{ or } 16$	A1 cao
		(2) [4]
2.	$32 = 2^5$ or $2048 = 2^{11}$, $\sqrt{2} = 2^{1/2}$ or $\sqrt{2048} = (2048)^{\frac{1}{2}}$	B1, B1
	$a = \frac{11}{2}$ (or $5\frac{1}{2}$ or 5.5)	B1
3.	$\left\{\frac{2}{\sqrt{12}-\sqrt{8}}\right\} = \frac{2}{\left(\sqrt{12}-\sqrt{8}\right)} \times \frac{\left(\sqrt{12}+\sqrt{8}\right)}{\left(\sqrt{12}+\sqrt{8}\right)}$	[3] M1
	$= \frac{\left\{2\left(\sqrt{12} + \sqrt{8}\right)\right\}}{12 - 8}$	A1
	$= \frac{2(2\sqrt{3} + 2\sqrt{2})}{12 - 8}$	B1 B1
	$= \sqrt{3} + \sqrt{2}$	A1 cso [5]

Question Number	Scheme					
4.	$x\sqrt{x} = x^{\frac{3}{2}}$	B1				
	$x^{-\frac{1}{2}} \to kx^{\frac{1}{2}}$ or $x^{\frac{3}{2}} \to kx^{\frac{5}{2}}$	M1				
	$x\sqrt{x} = x^{-1/2}$ $x^{-1/2} \to kx^{1/2}$ or $x^{3/2} \to kx^{5/2}$ $(y =) \frac{5x^{1/2}}{1/2} \dots + \frac{x^{5/2}}{5/2} (+C)$	A1 A1				
	$35 = \frac{5 \times 4^{\frac{1}{2}}}{\frac{1}{2}} + \frac{4^{\frac{5}{2}}}{\frac{5}{2}} + C$	M1				
	$C = \frac{11}{5} \qquad \text{or equivalent} 2\frac{1}{5}, 2.2$	A1				
	$y = 10x^{\frac{1}{2}} + \frac{2x^{\frac{5}{2}}}{5} + \frac{11}{5}$ (or equivalent <u>simplified</u>)	A1ft				
5 (a)	Translation parallel to x-axis	[7]				
5. (a)	Top branch intersects +ve y-axis Lower branch has no intersections No obvious overlap	M1 A1				
	$\left(0,\frac{3}{2}\right)$ or $\frac{3}{2}$ marked on y- axis	B1				
(b)	x = -2, y = 0	(3) B1 B1				
		(2) [5]				
6.	$b^2 - 4ac$ attempted, in terms of p . $(3p)^2 - 4p = 0$ or equivalent Attempt to solve for p e.g. $p(9p-4) = 0$					
	Must potentially lead to $p = k, k \neq 0$ $p = \frac{4}{9}$ (Ignore $p = 0$, if seen)					
		[4]				

Question Number	Scheme						
7. (a)	Discriminant: $b^2 - 4ac = (k+3)^2 - 4k$ or equivalent	M1 A1 (2)					
(b)	$(k+3)^2 - 4k = k^2 + 2k + 9 = (k+1)^2 + 8$						
(c)	For real roots, $b^2 - 4ac \ge 0$ or $b^2 - 4ac > 0$ or $(k+1)^2 + 8 > 0$	(2) M1					
	$(k+1)^2 \ge 0$ for all k , so $b^2 - 4ac > 0$, so roots are real for all k (oe)	A1 cso (2) [6]					
8. (a)	$4x - 5 - x^2 = q - (x - p)^2$, p, q are integers.						
	$\begin{cases} 4x - 5 - x^2 = \\ = -1 - (x - 2)^2 - [x^2 - 4x + 5] = -[(x - 2)^2 - 4 + 5] = -[(x - 2)^2 + 1] \end{cases}$	M1					
	$z - 1 - (\lambda - 2)$	A1 A1 (3)					
(b)	$ \left\{ "b^2 - 4ac" = \right\} 4^2 - 4(-1)(-5) \qquad \left\{ = 16 - 20 \right\} $ $ = -4 $						
	= -4	A1 (2)					
(c)	$y ightharpoonup$ Correct \cap shape	M1					
	Maximum within the 4 th quadrant	A1					
	Curve cuts through -5 or $(0, -5)$ marked on the y-axis	B1					
	- 5						
		(3) [8]					

Question Number	Scheme	Marks
9. (a)	a + 17d = 25 o.e., $a + 20d = 32.5$ o.e.	B1 B1
(b)	3d = 7.5 so $d = 2.5$	(2) M1
, ,	$a = 32.5 - 20 \times 2.5$ so $a = -17.5$ (*)	A1 cso
	и	(2)
(c)	$2750 = \frac{n}{2} \left[-35 + \frac{5}{2} (n-1) \right]$	M1 A1ft
	$\{ 4 \times 2750 = n(5n - 75) \}$	
	$4 \times 550 = n(n-15)$	M1
	$n^2 - 15n = 55 \times 40 \tag{*}$	A1 cso
(d)	$n^2 - 15n - 55 \times 40 = 0$ or $n^2 - 15n - 2200 = 0$	(4) M1
(u)	$(n-13n-33\times 40=0 01 n-13n-2200=0$ $(n-55)(n+40)=0 \qquad n=\dots$	M1
	n = 55 (ignore - 40)	A1
	$\frac{n-33}{n-33}$ (ignore - 40)	(3)
10 (a)	$(\frac{1}{2}, 0)$	B1 (1)
10. (a)	$\left(\begin{array}{c} 2, 0 \\ 1 \end{array}\right)$	
(0)	$\left(\frac{1}{2}, 0\right)$ $\frac{\mathrm{d}y}{\mathrm{d}x} = x^{-2}$	M1 A1
	At $x = \frac{1}{2}$, $\frac{dy}{dx} = \left(\frac{1}{2}\right)^{-2} = 4$ (= m)	A1
	Gradient of normal $= -\frac{1}{m}$ $\left(= -\frac{1}{4}\right)$	M1
	Equation of normal: $y - 0 = -\frac{1}{4} \left(x - \frac{1}{2} \right)$	M1
	2x + 8y - 1 = 0 (*)	Alcso (6)
(c)	$2 - \frac{1}{x} = -\frac{1}{4}x + \frac{1}{8}$ $= 2x^{2} + 15x - 8 = 0$ $(2x - 1)(x + 8) = 0$ leading to $x =$	M1
	$= 2x^{2} + 15x - 8 = 0 $ or $[8y^{2} - 17y = 0]$	
	F 4 7	M1
	$x = \left\lfloor \frac{1}{2} \right\rfloor$ or -8	A1
	$y = \frac{17}{8}$ (or exact equivalent)	A1ft (4)
	O	[11]

Question Number	Scheme	Marks
11. (a)	x = 2: $y = 8 - 8 - 2 + 9 = 7$ (*)	B1
		(1)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 4x - 1$	M1 A1
	$x = 2$: $\frac{dy}{dx} = 12 - 8 - 1 (= 3)$	A1ft
	y-7=3(x-2), $y=3x+1$	M1, <u>A1</u> (5)
	1	(3)
(c)	$m = -\frac{1}{3}$ (for $-\frac{1}{m}$ with their m)	B1ft
	$3x^2 - 4x - 1 = -\frac{1}{3}$, $9x^2 - 12x - 2 = 0$ or $x^2 - \frac{4}{3}x - \frac{2}{9} = 0$ (o.e.)	M1, A1
	$\left(x = \frac{12 + \sqrt{144 + 72}}{18}\right) \left(\sqrt{216} = \sqrt{36}\sqrt{6} = 6\sqrt{6}\right) \text{ or}$	M1
	$(3x-2)^2 = 6 \Rightarrow 3x = 2 \pm \sqrt{6}$	
	$(3x-2)^2 = 6 \rightarrow 3x = 2 \pm \sqrt{6}$ $x = \frac{1}{3}(2+\sqrt{6})$ (*)	A1cso
		(5)
		[11]

Examiner reports

Question 1

Overall this question was done poorly, with very few candidates scoring full marks. There were, however, many correct answers to part (a). Candidates were usually able to deal with either the negative part of the power or the fractional part, but some had problems in dealing with both. There were also some who did not understand the significance of the negative sign in front of the $\frac{1}{4}$ and the fact that it implied a reciprocal. Some assumed it implied a negative final answer, e.g. -2.

Part (b) was successfully completed by very few candidates, with the vast majority of errors being caused by the failure to raise 2 to the power 4. The power of the x inside the bracket was also often incorrectly calculated. It was common for candidates to multiply by x before raising to the power and so ending up with either $2x^3$ or $16x^3$. Others added the powers $-\frac{1}{4}$ and 4. Even when the bracket was correctly expanded, the extra x was often omitted or not combined with the other term.

Question 2

Many candidates could not deal with this test of indices. Two simple properties of indices were required: that a square root leads to a power of $\frac{1}{2}$ and the rule for adding the powers when multiplying. Those who identified these usually made good progress but the remainder struggled. Some re-wrote $32\sqrt{2}$ as $\sqrt{64}$ and then obtained a = 3 whilst others did obtain $\sqrt{2048}$ but usually failed to identify 2048 as 2^{11} . A number of candidates tried to use logs but this approach was rarely successful.

Question 3

This question proved discriminating with just under a half of the candidature gaining all 5 marks. About a third of the candidature gained only two marks by either correctly rationalising the denominator or by correctly simplifying $\sqrt{12}$ and $\sqrt{8}$.

A significant number of candidates answered this question by multiplying $\frac{2}{\sqrt{12}-\sqrt{8}}$ by

 $\frac{\left(\sqrt{12} + \sqrt{8}\right)}{\left(\sqrt{12} + \sqrt{8}\right)}$ to obtain $\frac{1}{2}\sqrt{12} + \sqrt{8}$ and proceeded no further. A small number of candidates

multiplied the denominator incorrectly to give either $12 + \sqrt{20} - \sqrt{20} - 8$ or $12 + \sqrt{72} - \sqrt{72} - 8$.

Those candidates who realised that $\sqrt{12} = 2\sqrt{3}$ and $\sqrt{8} = 2\sqrt{2}$ usually obtained the correct answer of $\sqrt{3} + \sqrt{2}$, although a few proceeded to the correct answer by writing $\frac{1}{2}\sqrt{12} + \sqrt{8}$ as $\sqrt{\frac{1}{4} \times 12} + \sqrt{\frac{1}{4} \times 8}$.

A number of candidates started this question by firstly simplifying $\sqrt{12}$ and $\sqrt{8}$ to give $\frac{2}{2\sqrt{3}-2\sqrt{2}}$, but many did not take the easy route of cancelling this to $\frac{1}{\sqrt{3}-\sqrt{2}}$ before

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rationalising. Although some candidates wrote $\frac{1}{\sqrt{3}-\sqrt{2}}$ as $\sqrt{3}-\sqrt{2}$, those candidates who rationalised this usually obtained the correct answer.

A small minority of candidates attempted to rationalise the denominator incorrectly by

multiplying
$$\frac{2}{\sqrt{12} - \sqrt{8}}$$
 by either $\frac{\left(\sqrt{12} - \sqrt{8}\right)}{\left(\sqrt{12} + \sqrt{8}\right)}$ or $\frac{\left(-\sqrt{12} + \sqrt{8}\right)}{\left(-\sqrt{12} + \sqrt{8}\right)}$.

Question 4

In this question the main problem for candidates was the integration of $x\sqrt{x}$, for which a

common result was
$$\frac{x^2}{2} \times \frac{2x^{\frac{3}{2}}}{3}$$
. Those who replaced $x\sqrt{x}$ by $x^{\frac{3}{2}}$ generally made good

progress, although the fractional indices tended to cause problems. Some differentiated instead of integrating. Most candidates used the given point (4, 35) in an attempt to find the value of the integration constant, but mistakes in calculation were very common. A significant minority of candidates failed to include the integration constant or failed to use the value of y in their working, and for those the last three marks in the question were unavailable.

Question 5

There were very mixed responses to this question; some answered it very well with neat sketches and clearly identified asymptotes but others appeared to have little idea. In part (a) most candidates knew that a translation was required and the majority knew it was horizontal and to the left. A few moved their graph up and a number of candidates ended up with graphs cutting both axes. Many identified the intersection with the *y*-axis as (0, 1.5) but some ignored this part of the question. It was clear in part (b) that a number of candidates still do not know the meaning of the term asymptote but many did give x = -2 and slightly fewer gave y = 0. However a number of candidates gave answers of y = -2 and x = 0, often despite having a correct dotted line on their sketch, and some gave other non-linear equations such as $y = \frac{3}{x}$.

Some candidates gave no answer to part (b) despite having the lines x = -2 and y = 0 clearly identified on their diagram.

Question 6

Some candidates opted out of this question but most realised that they needed to use the discriminant and made some progress. The most common error was to simplify $(3p)^2$ to $3p^2$ but there were many correct equations seen and usually these led to a correct answer. Many candidates chose to solve their equation by cancelling a p. In this case that was fine, since they were told that p was non-zero, but this is not good practice in general and factorising $9p^2 - 4p$ to p(9p - 4) is recommended.

Question 7

Candidates were required to give $(k+3)^2 - 4k$ as their answer to part (a). Any x terms included resulted in zero marks. Some candidates tried to solve $(k+3)^2 - 4k = 0$ and this was also not given any credit in this part.

Most candidates managed to complete the square correctly in part (b) and those starting with $k^2 + 2k + 9$ usually arrived at the correct answer for this part. However, several left their solution as $(k+3)^2 - 4k$, thus gaining no credit.

Part (c) was poorly done, with a substantial minority of the candidates not understanding what the question required. Quite a few realised that the determinant had to be greater than zero, but didn't know how to show this. M1 A0 was a common mark for those who tried a number of values for k. Candidates were expected to use their completion of the square and to argue that $(k+1)^2 \ge 0$ for all values of k.

A large minority were intent on trying to solve $k^2 + 2k + 9 = 0$, and concluded that there were no real roots. They demonstrated some confusion between the values of k and the information provided by the discriminant. Good candidates scored full marks on this question.

Ouestion 8

This question was poorly answered with about 10% of the candidature gaining all 8 marks.

In part (a), the $-x^2$ term and the order of the terms in $4x - 5 - x^2$ created problems for the majority of candidates. The most popular (and most successful) method was to complete the square. A large number of candidates struggled to deal with the $-x^2$ term with bracketing errors leading to incorrect answers such as $-(x + 2)^2 - 9$ or $-(x - 2)^2 - 9$. A successful strategy for some candidates involved negating the quadratic to get $x^2 - 4x + 5$ which was usually manipulated correctly to $(x - 2)^2 + 1$. Whilst a good number negated this correctly to $-(x - 2)^2 - 1$, some candidates wrote down incorrect results such as $-(x + 2)^2 - 1$, $(x + 2)^2 - 1$ or $-(x - 2)^2 + 1$.

Whilst a number of candidates stopped after multiplying out $q - (x - p)^2$, those who attempted to use the method of equating coefficients were less successful.

In part (b), most candidates wrote down $b^2 - 4ac$ for the discriminant and the majority achieved the correct answer of -4, although some incorrectly evaluated $4^2 - 4(-1)(-5)$ as 36. The most common error was for candidates to substitute the incorrect values of a = 4, b = 5 and c = -1 in $b^2 - 4ac$. Those candidates who applied the quadratic formula gained no credit unless they could identify the discriminant part of the formula.

In part (c), a majority of candidates were able to draw the correct shape of the graph and a number correctly identified the y-intercept as -5. Only a small minority were able to correctly position the maximum turning point in the fourth quadrant and some labelled it as (2, -1). A number of correct sketches followed from either candidates using differentiation or from candidates plotting points from a table of values. A number of candidates after correctly identifying the discriminant as -4, (including some who stated "that this meant no roots") could not relate this information to their sketch with some drawing graphs crossing the x-axis at either one or two points. Common errors included drawing graphs of cubics or positive quadratics, drawing negative quadratics with a maximum at (5, 0) or with a maximum at (0, -5).

Question 9

Although most candidates made a reasonable attempt at this question, only those who demonstrated good skills in algebra managed to score full marks.

The structure of parts (a) and (b) was intended to help candidates, but when the initial strategy was to write down (correctly) 3d = 32.5 - 25, there was sometimes confusion over what was required for the two equations in part (a). Even when correct formulae such as $u_{18} = a + 17d$ were written down, the substitution of $u_{18} = 25$ did not always follow. The work seen in these first two parts was often poorly presented and confused, but credit was given for any valid method of obtaining the values of d and a without assuming the value of a.

In part (c), many candidates managed to set up the correct sum equation but were subsequently let down by poor arithmetic or algebra, so were unable to proceed to the given quadratic equation. Being given 55×40 (to help with the factorisation in the last part of the question) rather than 2200 sometimes seemed to be a distraction.

Despite being given the 55×40 , many candidates insisted on using the quadratic formula in part (d). This led to the problem of having to find the square root of 9025 without a calculator, at which point most attempts were abandoned.

Question 10

This was a substantial question to end the paper and a number of candidates made little attempt beyond part (a). Part (c) proved quite challenging but there were some clear and succinct solutions seen.

Some stumbled at the first stage obtaining x = 2 or even 1 instead of $\frac{1}{2}$ to the solution of $2 - \frac{1}{x} = 0$ but most scored the mark for part (a).

The key to part (b) was to differentiate to find the gradient of the curve and most attempts did try this but a number had $-x^{-2}$. Some however tried to establish the result without differentiation and this invariably involved inappropriate use of the printed answer. Those who did differentiate correctly sometimes struggled to evaluate $\left(\frac{1}{2}\right)^{-2}$ correctly. A correct "show that" then required clear use of the perpendicular gradient rule and the use of their answer to part (a) to form the equation of the normal. There were a good number of fully correct solutions to this part but plenty of cases where multiple slips were made to arrive at the correct equation.

Most candidates set up a correct equation at the start of part (c) but simplifying this to a correct quadratic equation proved too challenging for many. Those who did arrive at $2x^2 + 15x - 8 = 0$ or $8y^2 - 17y = 0$ were usually able to proceed to find the correct coordinates of *B*, but there were sometimes slips here in evaluating $2 - \frac{1}{-8}$ for example.

There were a few candidates who used novel alternative approaches to part (c) such as substituting xy for 2x - 1 from the equation of the curve into the equation of the normal to obtain the simple equation 8y + xy = 0 from whence the two intersections y = 0 and x = -8 were obtained.

Question 11

A number of partial attempts at this question may suggest that some were short of time although the final part was quite challenging.

Most secured the mark in part (a) although careless evaluation of $2 \times (2)^2$ as 6 spoiled it for some. Apart from the few who did not realise the need to differentiate to find the gradient of the curve, and hence the tangent, part (b) was answered well. Some candidates though thought that the coefficient of x^2 (the leading term) in their derivative gave them the gradient. There was the usual confusion here between tangents and normals with some candidates thinking that $\frac{dy}{dx}$ gave the gradient of the normal not the tangent. In part (c) many knew they needed to use the perpendicular gradient rule but many were not sure what to do. A common error was to find the equation of a straight line (often the normal at P) and then attempt to find the intersection with the curve. Those who did embark on a correct approach usually solved their quadratic equation successfully using the formula, completing the square often led to difficulties with the x^2 term, but a few provided a correct verification.

Statistics for C1 Practice Paper Gold Level G4

Mean score for students achieving grade:

	Max	Modal	Mean								
Qu	score	score	%	ALL	A *	Α	В	С	D	Е	U
1	4		57	2.29	3.76	3.36	2.81	2.42	2.26	1.86	1.51
2	3		42	1.26		2.27	1.47	1.09	0.82	0.60	0.33
3	5		67	3.37	4.83	4.47	3.90	3.48	3.12	2.75	1.91
4	7		57	4.01		6.60	5.45	4.75	3.45	2.96	1.48
5	5		49	2.44		4.16	3.07	2.39	1.84	1.39	0.76
6	4		58	2.33		3.63	3.05	2.58	1.97	1.33	0.55
7	6		45	2.70	5.28	4.49	3.33	2.78	2.22	1.71	0.68
8	8		52	4.19	7.58	6.54	5.19	4.38	3.65	2.98	1.66
9	11		54	5.94		10.09	8.28	6.79	5.24	4.23	2.48
10	11		47	5.12	10.95	10.36	8.63	6.58	4.23	3.19	1.09
11	11		50	5.47		9.61	7.24	5.54	4.03	2.69	1.15
	75		52	39.12		65.58	52.42	42.78	32.83	25.69	13.60